deformation. The ratio of the production efficiencies α_f/α_u decreases with increasing strain.

2. There is a theoretical possibility and an experimental indication for a larger efficiency to exist during the first cycles of cyclic loading than during the initial stage of unidirectional deformation.

3. The shear-modulus changes observed during cyclic stressing can be attributed to the formation of dislocation dipoles from dislocations of the initial network.

To detail the exact mechanisms it is necessary to perform resistivity measurements, shear-modulus measurements, and electron-microscopy studies, and correlate these for the same sample.

JOURNAL OF APPLIED PHYSICS VOLUME 39, NUMBER 7 JUNE 1968

Third-Order Elastic Constants of Single-Crystal and Polycrystalline Columbium

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(Received 8 January 1968)

The second-order and third-order elastic constants for both single-crystal and polycrystalline columbium were determined at 298°K using an ultrasonic phase comparison method. It is shown that any dislocation contribution to the measurements is negligible with respect to other uncertainties. The third-order elastic constants (Brugger convention) in units of 10^{12} dyn/cm^2 are

$c_{111} = -25.64 \pm 0.25$; $c_{144} = -3.43 \pm 0.10$; $\nu_1 = -4.8 \pm 1.2$;

$c_{112} = -11.40 \pm 0.25$; $c_{166} = -1.677 \pm 0.05$; $\nu_2 = -3.70 \pm 0.2$;

$c_{123} = -4.67 \pm 0.25$; $c_{456} = +1.366 \pm 0.05$; $\nu_3 = +0.75 \pm 0.05$.

The Grüneisen parameter calculated from these values is 1.511 ± 0.026 for the single crystal, and 1.546 ± 0.092 for the polycrystal, in good agreement with the value 1.52 calculated from bulk-property measurements. The polycrystalline third-order elastic constants are in very good agreement with those calculated from the single-crystal values by the method of Chang.

I. INTRODUCTION

Of the several methods of determining the third-order elastic constants (TOEC) of solids,[1] the one that has proven most useful involves the measurement of the dependence of ultrasonic wave velocities on a static bias stress. Most of the early work in this area was restricted to the use of hydrostatic pressure as the bias stress[2,3] to attain stresses high enough so that the dependence was measurable without introducing plastic flow in the samples. This method cannot provide enough information to obtain all of the TOEC, however. In an elastically isotropic sample, for example, there are only two nondegenerate pure-mode elastic waves which can be propagated, providing two relations involving the three independent TOEC. In crystals of cubic symmetry, there are three pure-mode waves for a given propagation direction and therefore three independent relations among the six independent TOEC. However, the results of these measurements are useful in theoretical studies of those anharmonic properties which are bulk properties in the sense that they do not depend on the crystal orientation, such as thermal expansion (or the Grueneisen parameter) in isotropic or cubic symmetry crystals.[4] The results of the hydrostatic pressure experiments give just the combinations of TOEC which are required in these studies, but they are limited in that they cannot provide the entire set of TOEC which is required to define completely the anharmonic nature of the sample.

Recent developments of more sensitive experimental methods of determining relative changes in ultrasonic wave velocities have allowed the extension of these methods to include the use of uniaxial bias stresses, as well as hydrostatic pressure.[5] It has been found, however, that in materials with a low yield stress such as pure copper, that at even very low values of uniaxial bias stress, the ultrasonic wave velocity will be affected by a change in the dislocation contribution to the dynamic second-order elastic moduli caused by the applied bias stress.[6] This change in the ultrasonic wave velocity due to dislocations is impossible to separate from the change due to intrinsic lattice anharmonicity, and therefore care must be taken to prevent this effect from occurring. Methods that have been used to pin

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